ACTIVE OPTICAL SYSTEMS

AN032

The Relationship between Inter-Actuator Throw and Resonance Frequency on Plate-Type Deformable Mirrors

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Revision: 7/11/11

Abstract

In this application note we show that there is an inverse relationship between the resonance frequency and the maximum inter-actuator throw for plate-type deformable mirrors.

Introduction

We often get requests for deformable mirrors that specify both a large inter-actuator stroke and a high resonance frequency. Unfortunately, this is not always possible due to physical limitations of the plate-type DM architecture and the materials we use in its construction. In this application note, we use basic plate theory to illustrate this relationship.

The basic architecture of a plate-type deformable mirror is an array of actuators bonded to a faceplate and a thicker base plate. For this analysis, we are only concerned with the faceplate bonded to actuators. One of the advantages of the plate-type DM architecture is that the first resonance is typically dominated by the portion of the faceplate that is supported between two actuators. Therefore the DM actuator count can be scaled without reducing the resonance frequency.

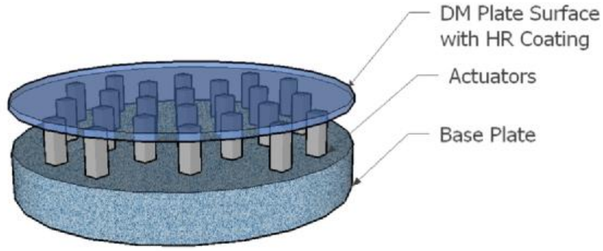

Figure 1 – Basic Plate-type DM Architecture

For a basic understanding of the relationship between the inter-actuator throw and the resonance frequency, we can analyze a section of faceplate spanning the gap between two actuators by simplifying the mechanical model of the DM faceplate to that of a clamped plate. Timoshenko provides analysis of plates with a variety of boundary conditions and applied force conditions.[1] For a circular plate with clamped edges, the maximum deflection for a point load at the center is given by,

$$w_{\max} = \frac{(3+\nu)a^2}{16\pi(1+\nu)D} P$$

where ν is Poisson's ratio, a is the radius, P is the applied force, and D is given by,

$$D = \frac{Eh^3}{12(1-\nu^2)}.$$

The maximum stress can be approximated by,

$$\sigma_{\max} = \frac{P}{h^2}\left[(1+\nu)\left(0.485\log\frac{a}{h}+0.52\right)+0.48\right].$$

We can approximate the first resonance using the classic formula derived from F=ma=kx given by,

$$f = 2\pi\sqrt{\frac{k}{m}}$$

where m is the mass is approximated by $\pi h a^2 \delta$, k is the ratio of force to deflection, and δ is the density of the faceplate.

Figure 2 shows these relationships for a silicon faceplate varying in thickness from 0.5 to 2.0 mm and a deflection for a 50 N force. We can see in these results that the increasing thickness reduces both the deflection and stress and increases the first resonance. We generally found the results to be consistent with our laboratory results, but the resonance frequency was significantly higher than our laboratory results.

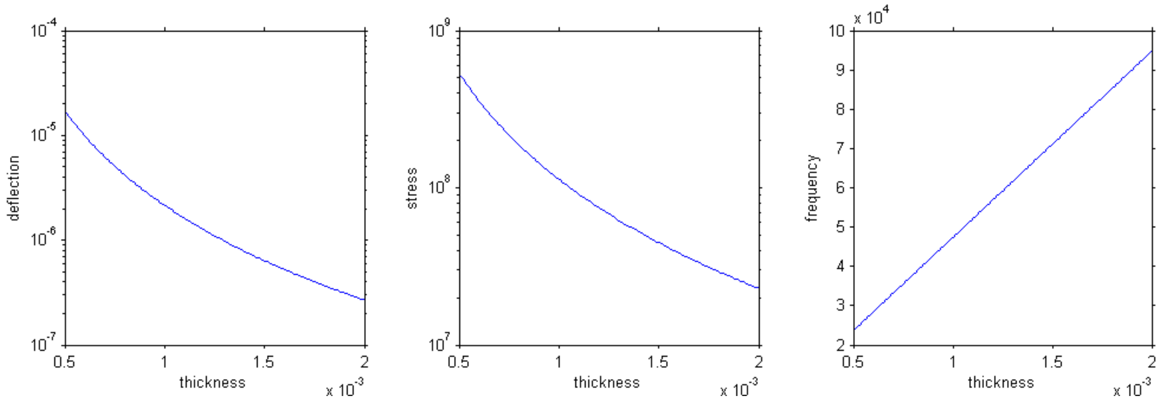

Figure 2 – Clamped plate deflection, stress, and resonance frequency as a function of plate thickness

The limit to the inter-actuator throw of a plate-type DM is the stress induced by this throw. This stress can result in fracture of the faceplate or the bonds holding the DM components together. The bond stress will be related to the faceplate stress but not exactly equal to it. We next found the deflection for a 20 MPa stress for each of the thicknesses. Figure 3 shows the deflection against the resonance frequency. This plot clearly shows that increasing the resonance frequency reduces the maximum inter-actuator throw.

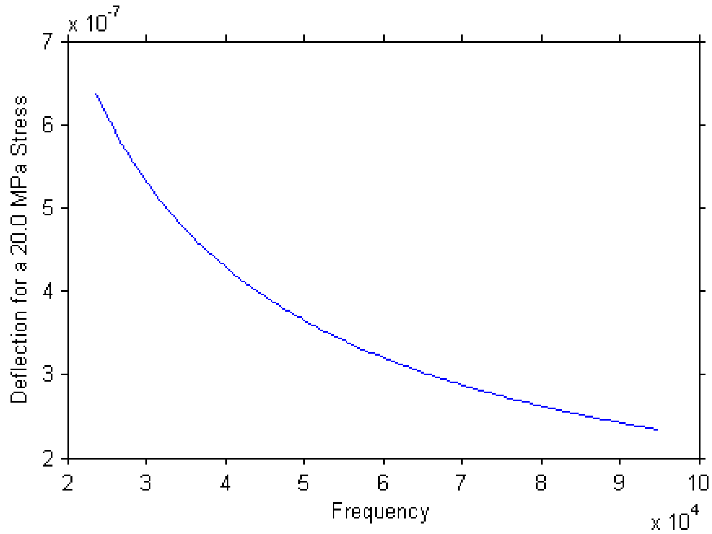

Figure 3 - Relationship between maximum deflection for a 20 MPa stress and the resonance frequency

Engineering Plate-Type DMs

There are both material and engineering parameters that can be used to adjust both the resonance frequency and the maximum inter-actuator throw. The maximum inter-actuator stroke for a given maximum faceplate stress is given by,

$$w_{\max} = \frac{(3+\nu)a^2}{16\pi(1+\nu)\left(\frac{Eh^3}{12(1-\nu^2)}\right)} \frac{\sigma_{\max} h^2}{\left[(1+\nu)\left(0.485\log\frac{a}{h}+0.52\right)+0.48\right]}.$$

$$\propto \frac{\sigma_{\max} a^2}{Eh}$$

The resonance frequency is expressed by,

$$f = 2\pi\sqrt{\frac{16\pi(1+\nu)\left[\frac{Eh^3}{12(1-\nu^2)}\right]}{\left((3+\nu)a^2\right)\pi h a^2 \delta}}.$$

$$\propto \frac{h\sqrt{E}}{\delta a^2}$$

In each of the equations above, we reduced the equation into most of the fundamental dependencies without the Poissions ratio term. This reduction is not perfectly accurate because of some of the terms that were eliminated, but is useful to illustrate the key parameters in the frequency The product of the dependences of the frequency and the maximum inter-actuator displacement is given by

$$f w_{\max} \propto \frac{h\sqrt{E}}{\delta a^2}\frac{\sigma_{\max} a^2}{Eh} = \frac{\sigma_{\max}}{\delta\sqrt{E}}.$$

Therefore, to increase both the resonance frequency and the throw simultaneously, the density and the modulus need to be decreased. The density decrease will obviously decrease the mass and increase the resonance frequency. The less obvious change is the decrease in the modulus. This decrease reduces the effective spring constant, which enables less force to be able to move the faceplate and generates less stress, but only reduces the resonance frequency by the square-root of the reduction. While it may be possible to find better materials for the DM faceplate, there are a limited number of materials that are suitable for other reasons like achieving a good polish and the ability to be coated for reflectivity.

Figure 4 shows a comparison of different materials. Table 1 shows the materials parameters used in the analysis. These materials were chosen because they illustrate the relationship derived above rather well. When comparing silicon and fused silica, the density is almost the same, but the modulus is 2.6 times larger for silicon. When comparing fused silica and carbon fiber, the modulus is nearly the same, but the density is reduced by a factor of 1.4. Carbon fiber, being the lowest density and the lowest modulus offers the best range of performance for a deformable mirror, but achieving other properties key to DM performance like surface polish and high quality coatings may be difficult.

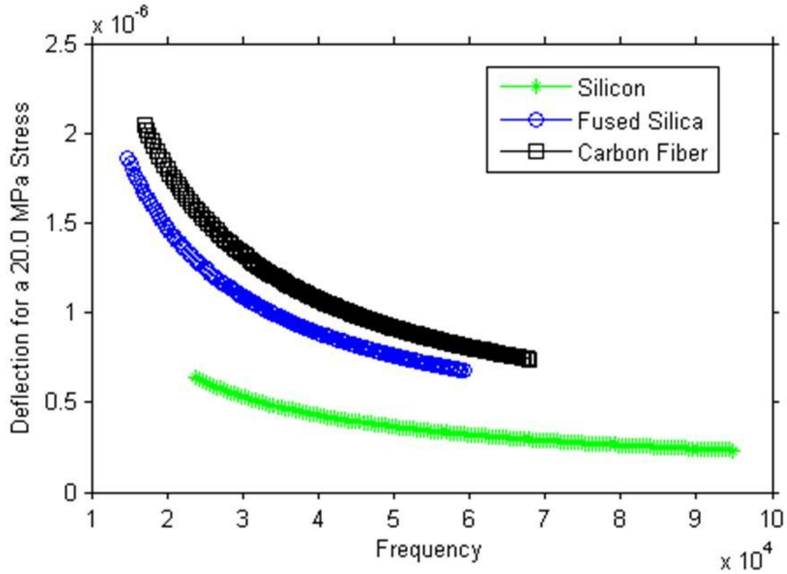

Figure 4 - Comparison of Deflection to Frequency Performance for Various Materials

Table 1- Key Materials Parameters for DM Analysis

Material	Modulus (GPa)	Poissions Ratio	Density (kg/m^3)
Silicon	185	0.28	2330
Fused Silica	72	0.17	2203
Carbon Fiber	70	0.1	1600

Conclusions

From this analysis we established that there was an inverse relationship between inter-actuator throw and resonance frequency for plate-type DMs. There are some engineering and material parameters that can affect both quantities, but the mechanics of the plate-type DM architecture prevent a realistic arbitrary control of both throw and resonance.

Appendix: Matlab Analysis Code

```
% study of inter-actuator limits
% Timoshenko - Circular Plate Loaded at the Center, p. 67-72
setup;
ppt=1;

hvec = [0.5:0.01:2.0] * 1e-3; %thickness
material = 1; materialLabel{1} = 'Silicon';
material = 2; materialLabel{2} = 'Fused Silica';
material = 3; materialLabel{3} = 'Carbon Fiber';
for material=1:length(materialLabel);
    if (ppt)
        TitleSlidePowerPoint(materialLabel{material});
    end;
    if (material==1) %silicon
        %wikipedia
        nu = 0.28;
        density = 2330; %2.33 g/cc =  2330 kg/m^3
        E = 185e9; %pascal
    elseif (material==2)
        %wikipedia
        nu = 0.17;
        density = 2203; %2.33 g/cc =  2330 kg/m^3
        E = 72e9; %pascal
    else
        %http://www.performance-
composites.com/carbonfibre/mechanicalproperties_2.asp
        nu = 0.1;
        density = 1600; %2.33 g/cc =  2330 kg/m^3
        E = 70e9; %pascal
    end;

    a = 6e-3;
    P = 50.0;
    delta = 1e-6;
    stressMax = 20e6;
    for ii=1:length(hvec);
        h = hvec(ii);
        D = (E .* h.^3) ./ (12 .* (1-nu.^2));
        stress(ii) = P./h^2 * ( (1+nu) * (0.485 * log(a./h) + 0.52) + 0.48);

        % clamped plate
        deflection(ii) = P / (16*pi*D) * a^2;
        k = P ./ deflection(ii);
        m = (pi * a^2 * h) * density;
        f(ii) = sqrt(k/m) / (2.0*pi);

        %reverse solve for P
        Pcalc = delta * k;
        stressCalc(ii) = Pcalc./h^2 * ( (1+nu) * (0.485 * log(a./h) + 0.52) +
0.48);

        %for a fixed stress, what is the throw
```

```
        Pcalc = stressMax .* h^2  ./ ( (1+nu) * (0.485 * log(a./h) + 0.52) +
0.48);
        deflectionCalc(ii) = Pcalc ./ k;

    end;

    %% summary 1
    nf([ 286         58        1055         630]);
    subplot(2,3,1);
    plot(deflection,stress);
    xlabel('deflection');
    ylabel('stress');

    subplot(2,3,2);
    plot(f,stress);
    xlabel('frequency');
    ylabel('stress');

    subplot(2,3,3);
    loglog(f,deflectionCalc);
    plot(f,deflectionCalc);
    xlabel('Frequency');
    ylabel(sprintf('Deflection for a %.1f MPa Stress',stressMax./1e6));

    subplot(2,3,4);
    semilogy(hvec,deflection);
    ylabel('deflection');
    xlabel('thickness');

    subplot(2,3,5);
    semilogy(hvec,stress);
    ylabel('stress');
    xlabel('thickness');

    subplot(2,3,6);
    plot(hvec,f);
    ylabel('frequency');
    xlabel('thickness');
    if (ppt)
        ToPPT()
    end;

    %% summary 2
    nf([156         354        1185         334]);
    subplot(1,3,1);
    semilogy(hvec,deflection);
    ylabel('deflection');
    xlabel('thickness');

    subplot(1,3,2);
    semilogy(hvec,stress);
    ylabel('stress');
    xlabel('thickness');

    subplot(1,3,3);
```

```
    plot(hvec,f);
    ylabel('frequency');
    xlabel('thickness');
    if (ppt)
        ToPPT()
    end;

    %% conclusion
    nf;
    plot(f,deflectionCalc);
    xlabel('Frequency');
    ylabel(sprintf('Deflection for a %.1f MPa Stress',stressMax./1e6));
    if (ppt)
        ToPPT()
    end;
    fSave{material}=f;
    deflectionSave{material}=deflectionCalc;
    close all;
end;

%% final conclusion
if (ppt)
    TitleSlidePowerPoint('Summary');
end;
nf;
for material=1:length(materialLabel);
    plot(fSave{material},deflectionSave{material},getLineSpec(material));
    hold on;
end;
xlabel('Frequency');
ylabel(sprintf('Deflection for a %.1f MPa Stress',stressMax./1e6));
legend(materialLabel,'Location','Best');
if (ppt) ToPPT(); end;
```

References

[1] S. Timoshenko and S. Woinowsky-Krieger, Theory of Plates and Shells, McGraw-Hill (1959).